EXPERIMENT 2

Density: A Study in Precision and Accuracy (Part A)

Introduction

In this lab, you will be investigating the concepts of precision and accuracy. You will be doing an experiment in which you will be measuring the density of some glass beads. Although you will be learning a little bit about density measurements, the primary goal of the lab is for you to come to grips with the separate concepts of accuracy and precision.

In any scientific investigation, when the results are reported, it is standard practice that the investigating scientist carefully consider both the issues of accuracy and precision. Since these two concepts are often confused, we will begin with a careful definition of each.

Precision: A measure of the amount of random variation in the measurement of data.

Accuracy: A measure of how far an experimental result is from the true or correct value.

Whenever a scientist makes measurements, there will always be some random variation in the values recorded. For example, if one were to use a stop watch to measure the time it took an object to fall a certain distance, one might record data such as the following:

1.49 s, 1.48 s, 1.53 s, 1.48 s, 1.50 s, 1.47 s, 1.52 s, 1.52 s, 1.46 s

The random variation does not necessarily reflect an error on the part of the person doing the measurements, but rather it may reflect the limit of the precision of the time measuring device (and the ability of the person controlling the stopwatch to hit the button at the right time). The precision of the experiment is a measure of the size of the random variation in the experiment. In this experiment, you will learn to calculate the standard deviation of the measurement: the most common accepted statistical measure of precision.

Another way of thinking about precision is to think of precision as a measure of the amount of random error in an experiment. Any experimental error is considered to be random if its result could make the calculated or measured value either too high or too low. Any potential error in an experiment which could have a predicted effect on the result, making it either definitely too high or too low, would be considered a systematic error (see below).

For the same experiment as described above, if the exact height from which the mass was dropped was known the equations of motion from the basic physics course, assuming the acceleration due to gravity to be 9.80 m/sec^2, used to calculate the theoretical time it should take the mass to fall. In this case, the correct theoretical value could be compared to the results. For example, if the "true" or "theoretical" value for the time it should take the mass to drop were 1.45 seconds, the accuracy of the experiment could be calculated. In this experiment, you will measure the accuracy as the %-error of the measurement.

In general, scientists are always able to measure the random error of an experiment. There are situations in which there is no known "true" or "accepted" value for a measurement. In this case, the scientist may not be able to calculate the accuracy as a %-error. Normally, in such a situation, the scientist will do a very similar experiment to the one to be reported, only doing the very similar experiment on slightly different case in which the "true" value is known. The scientist will then calculate the %-error in this case as a check on the validity of the measurement they are reporting.

Another way of thinking of accuracy is to think of accuracy as a measure of the systematic error in an experiment. For example, in the mass-dropping experiment described above, if one were to anticipate the mass hitting the floor, and press the button just a little before the mass hit, that would definitely make the time measured be too small. This would be an example of a systematic error. If the person doing the experiment were to consistently make such an error, it would effect the accuracy (and therefore the %-error), but not the precision (and therefore not the standard deviation).

Theory

For a set of measurements of a variable x, the standard deviation is calculated using the following equation:

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}} \qquad \textbf{(EQ 2.1)}$$

Where σ is the symbol for standard deviation (sometimes the letter s is used to represent the standard deviation). The symbol x represents the individual measurements, while the symbol $\bar{x}$ represents the average of the measurements in question. In this equation, n is the number of measurements. For example, one could calculate the standard deviation of the measurements of time listed above. The calculation is shown below:

TABLE 2.1

t	$t - \bar{t}$	$(t - \bar{t})^2$
1.49	–0.004	0.000016
1.48	–0.014	0.000196
1.53	+0.036	0.001296
1.48	–0.014	0.000196
1.50	+0.006	0.000036
1.47	–0.024	0.000576
1.52	+0.026	0.000676
1.52	+0.026	0.000676
1.46	–0.034	0.001156

$$t = \frac{13.45}{9} = 1.494 \text{ s} \qquad \sum (t - \bar{t})^2 = 0.004824 \text{ s}^2 \qquad \textbf{(EQ 2.2)}$$

Therefore, the standard deviation is:

$$\sigma = \sqrt{\frac{0.004824}{9-1}} = 0.0245 \text{ s} \qquad \textbf{(EQ 2.3)}$$

The average and the standard deviation are combined to yield the result that the time it took the mass to fall is:

$$t = 1.494 \pm 0.025 \text{ s} \qquad \textbf{(EQ 2.4)}$$

Note that both the average value and the standard deviation should end with the same number of digits past the decimal. Also note that the units label follows the standard deviation, not the average value.

The accuracy of the time measurement may be calculated as well. The equation for %-error is as follows:

$$\%\ \text{error} = \frac{\text{measured value} - \text{theoretical value}}{\text{theoretical value}} \times 100\ \% \qquad \textbf{(EQ 2.5)}$$

If the measured value is below the accepted or theoretical value, then there will be a negative %-error.

For the time experiment described above, then, the %-error is calculated (assuming the true value to be 1.45 s) as:

$$\%\ \text{error} = \frac{1.494 - 1.45}{1.45} \times 100\ \% = 3.0\ \% \qquad \textbf{(EQ 2.6)}$$

In general, a %-error is only reported to one or at most two significant figures.

In this experiment, you will be measuring both the mass and the volume of some glass beads. The density will be measured using the well-known equation:

$$d = \frac{m}{v} \qquad \textbf{(EQ 2.7)}$$

After calculating both the mass and the volume of some glass beads a number of times, you will calculate the average values and the standard deviations of both values. You will then calculate the density, the standard deviation of the density, and the %-error from the true value for the density of the glass beads.

What about the precision of your density measurement? In other words, you will be measuring the standard deviation of the mass measurement as well as of the volume measurement. The precision of the mass and volume measurements are determined when the standard deviation is measured. The question is how to use these results to estimate the size of the random error (precision) in the density when it is calculated by dividing the mass by the volume.

One way to do this would be to simply calculate the random error of the two measurements used to calculate the density and add the two errors. There are two problems with this approach. First, one cannot add apples to oranges. In other words one cannot add the error in the mass (with units grams) to the error in the volume (with units of milliliters). A partial solution to this problem would be to calculate the percentage of the two errors and adding them. In other words, if the standard deviation of the mass measurement was 5.2% of size of the average mass, and the standard deviation of the volume measurement was 8.4% of the size of the average volume, one could conclude that the random error in the density measurement was 5.2 + 8.4 = 13.6%.

The problem with this solution is that it overestimates the error. There is a significant probability that the error in the volume will have the opposite effect on the density as the error in the mass measurement. In other words, the errors can cancel. The correct statistical measurement of the combined error due to two measurements (*x* and *y*) being used to calculate a secondary value (*z*) is given by the following equation:

$$z = \frac{x}{y} \qquad \text{or} \qquad z = xy \qquad \textbf{(EQ 2.8)}$$

The estimated error, Δz, can be calculated as follows,

$$\Delta z = z\sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2} \qquad \textbf{(EQ 2.9)}$$

Where z is the calculated value and Δz is the estimated error in the calculated value. In this formula, σ_x and σ_y are the standard deviation in the measured values *x* and *y*. Therefore, the final result for the calculated value is $\bar{z} \pm \Delta z$.

Experimental Procedure

PART I Measuring the mass of the glass beads

Using a balance, measure the mass of forty glass beads eight different times. It is most convenient to measure the beads in a weighing boat. Be sure to measure different beads each time to ensure that you are getting a random selection of the beads in the jar. Record the eight values of the mass in the table below, being sure to get as much precision out of the balance as possible. After recording the values, determine the average mass if the beads. Go on to calculate the standard deviation of the mass measurements. In addition, report the average ± the standard deviation. Be sure to report the units and the number of significant figures properly. Show all your calculations.

PART II Measuring the volume of the glass beads

Fill a buret about one-third to one-half full with deionized water. Carefully record the volume of water in the buret. If you are not sure how to record the volume measurement from a buret accurately to two places past the decimal, ask your instructor for help.

Now, add forty glass beads to the buret and record the volume again. Repeat this process for a total of eight additions of forty beads, recording the final volume each time. Record the results in the table below. When you are done, drain the water from the buret and dump the wet beads into the container provided marked “wet glass beads”. Calculate the volume of forty beads for each of the eight cases, and continue on to calculate the standard deviation of the volume measurements. Report the average of the volume of forty beads ± the standard deviation in the correct format.

PART III Calculating the density and the error in the density.

- First, use the average mass and average volume from above to calculate the density of the glass beads.
- Next, calculate the %-error of your measurement, assuming that the correct value for the density of the glass beads is d = 2.35 g/ml.
- In addition, calculate the random error in the density using Equation 2.9 for calculating the accumulated uncertainty for a calculation involving two measurements.
- Finally, record the density of the glass beads from your measurements as $d = \bar{d} \pm \Delta d$

Post Lab Questions

1. Is the size of your random error big enough to explain the difference between your measured value of density and the expected value of 2.35 g/ml? Explain.

2. Based on your answer to question #1, would you need to invoke some sort of systematic error to explain the difference between your calculated density and the expected value of 2.35 g/ml, or can all the error be assumed to be due to random error? Explain.

3. Give two examples of random error in this experiment.

4. Give two examples of systematic error in this experiment. In each case, would the proposed systematic error make the calculated density too high or too low when compared to the correct value?